Province of the
EASTERN CAPE
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

SEPTEMBER 2018

MATHEMATICS P2

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages, including 1 page information sheet, and an answer book of 21 pages.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.

2. Answer ALL the questions in the ANSWER BOOK provided.

3. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.

4. Answers only will NOT necessarily be awarded full marks.

5. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.

6. If necessary, round off answers to TWO decimal places, unless stated otherwise.

7. Diagrams are NOT necessarily drawn to scale.

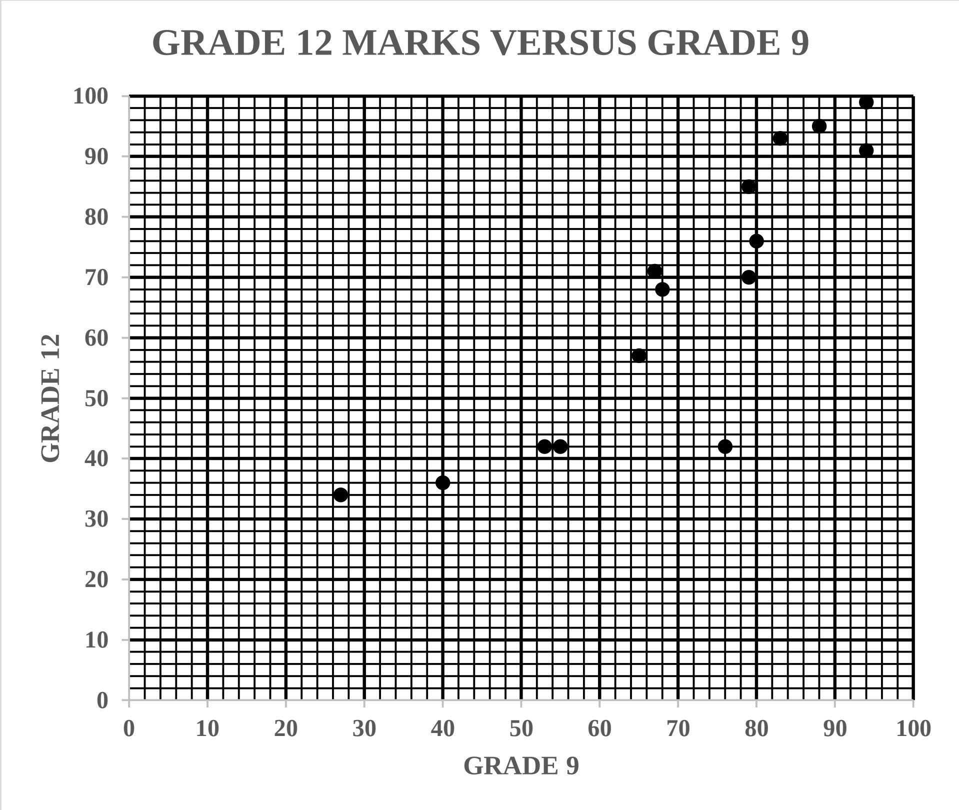
8. An information sheet with formulae is included at the end of the question paper.

9. Write neatly and legibly.

QUESTION 1

Every year the Grade 9s have to choose whether to take Mathematics or Mathematical Literacy. 15 learners were selected at random and their marks as a percentage for their Grade 9 report and their Grade 12 final exam are recorded in a table.

Grade 9	27	40	53	55	65	67	68	76	79	79	80	83	88	94	94
Grade 12	34	36	42	42	57	71	68	42	70	85	76	93	95	99	91



1.1 Determine the equation of the least squares regression line. (3)

1.2 If a learner in Grade 9 final exams obtains 41%, estimate what mark in percentage did he get in Grade 12. [Round off your answer to the nearest integer] (2)

1.3 Draw the least squares regression line on the scatter plot (2)

1.4 Calculate the correlation coefficient. (2)

1.5 Is the Grade 9 report mark a good predictor of the final matric mark? Motivate your answer. (2)

[11]

QUESTION 2

The number of learners absent from 11 weekend classes in a year were recorded as follows.

10 13 15 17 18 23 24 26 28 28 29

2.1 Determine the range of the above data. (1)

2.2 Calculate the average number of the learners absent from a weekend class. (2)

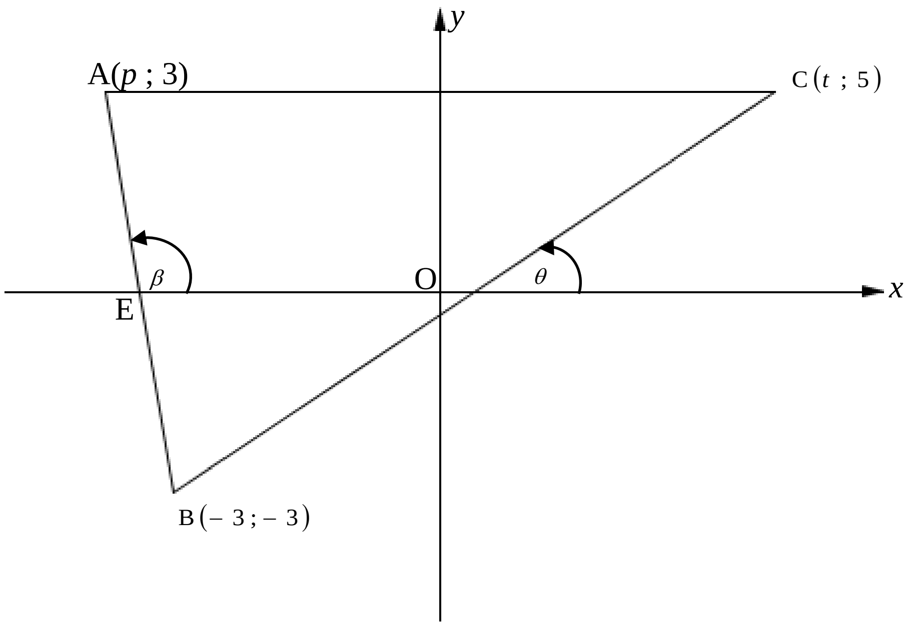
2.3 Calculate the standard deviation of the above data (3)

2.4 Determine the number of weeks where the attendance of the learners lies outside one standard deviation from the mean. (2)

[8]

QUESTION 3

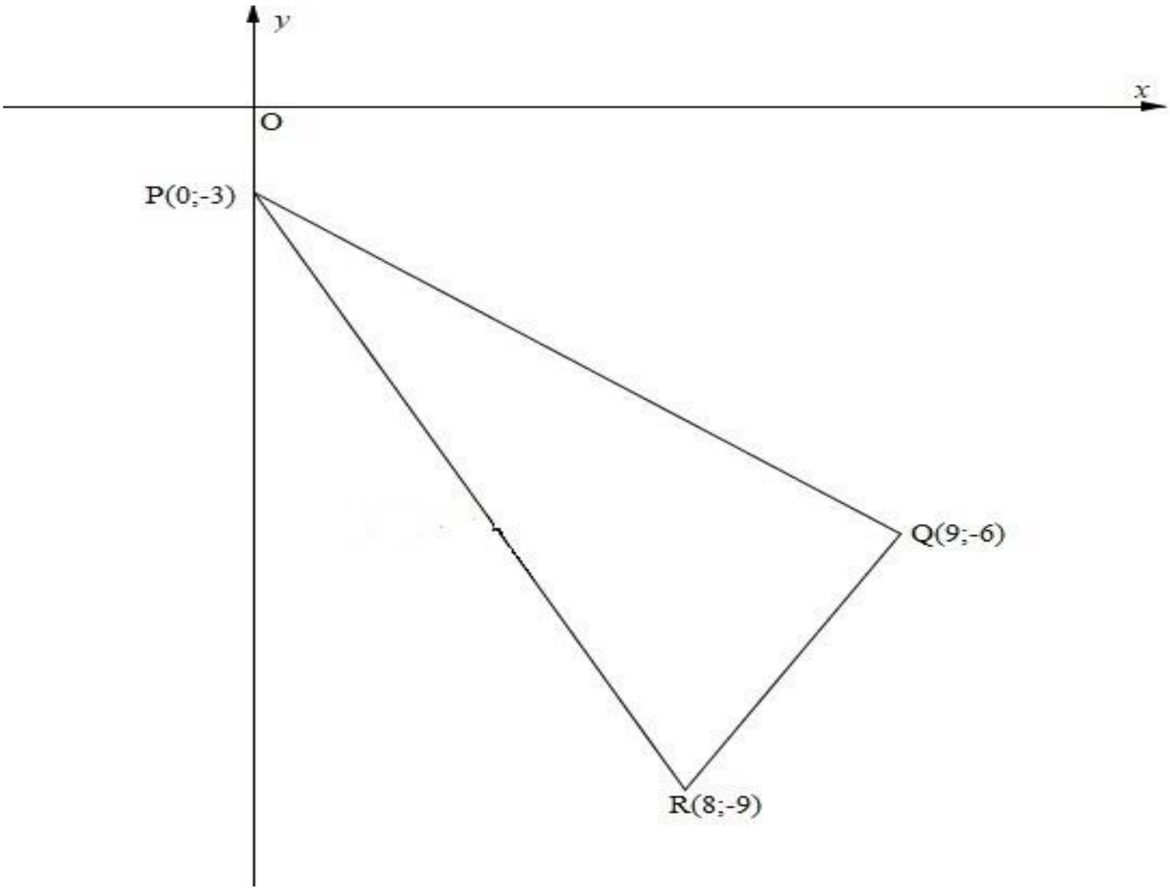
In the diagram below, points A (p; 3), $B(-3;-3)$ and $C(t;5)$ are the vertices of triangle ABC. The length of $BC = \sqrt{89}$ units. The equation of line AB is $y = -3x - 12$. E is the x-intercept of line AB.



3.1	Calculate the value of t	(5)
3.2	Calculate the value of p	(4)
3.3	Determine the coordinates of E	(2)
3.4	Determine the coordinates of M, the midpoint of AC if the value of t = 2 and p = -5.	(2)
3.5	Why is $EM \parallel BC$?	(4)
3.6	Calculate the size of $A\hat{B}C$	(4)
		[21]

QUESTION 4

In the diagram below, $P(0;-3)$, $Q(9;-6)$ and $R(8;-9)$ are the vertices of a triangle in the Cartesian plane.



4.1 Determine the length of PR (2)

4.2 Determine the co-ordinates of the midpoint of PR (2)

4.3 Show that $P\hat{Q}R = 90^0$ (4)

4.4 Determine the equation of the circle passing through P, Q and R (2)

4.5 Determine the equation of the tangent to the circle P, Q and R passing through point P. (5)

4.6 T is a point with co-ordinates $T(\cos\theta;\sin\theta)$, and the distance between T and R is $\sqrt{146}$ units, determine the value of $\tan\theta$. (5)

[20]

QUESTION 5

5.1 If $\sin A = -\frac{3}{7}$, where $A \in [90^0 ; 270^0]$ determine, using a diagram, without the use of a calculator, the value of $\sin(A + 30^0)$. (4)

5.2 Simplify fully to one trigonometric ratio of *x*:
$-\sin^2(90^0 - x) - \tan x.\cos(-x).\sin(-x - 360^0)$ (6)

5.3 Show that, for any given value of $\hat{A}$, the roots of the equation $x^2 - 2x\sin A = \cos^2 A$ are real and rational. (3)

5.4 Prove the identity:
$$\frac{\cos 3x}{\sin x} + \frac{\sin 3x}{\cos x} = \frac{2}{\tan 2x}$$ (3)

5.5 If $\cos 22^0 = p$ and $\sin 38^0 = q$, without the use of a calculator, determine the following in terms of *p* and *q*.

5.5.1 $\sin 68^0$ (2)

5.5.2 $\cos 16^0$ (4)

[22]

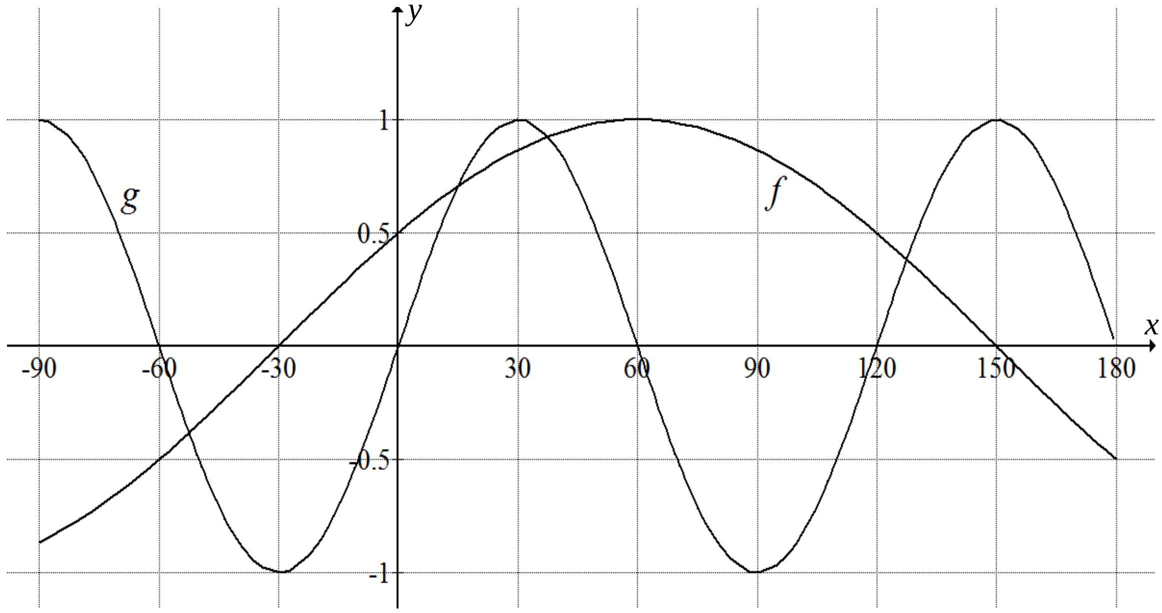
QUESTION 6

6.1 A function is defined as $f(x) = a\cos(x - p) + 1$.
The function satisfies the following conditions:

- The period is 360^0
- The range is $y \in [-1; 3]$
- The co-ordinates of a maximum point are $[210^0; 3]$

Write down the values of *a* and *p*. (2)

6.2 In the diagram below, the functions $f(x) = \cos(x - 60^0)$ and $g(x) = \sin 3x$ are drawn for $x \in [-90^0; 180^0]$.



For what value(s) of *x* is

6.2.1 $f'(x) = 0$, where $x \in [-90^0; 180^0]$? (1)

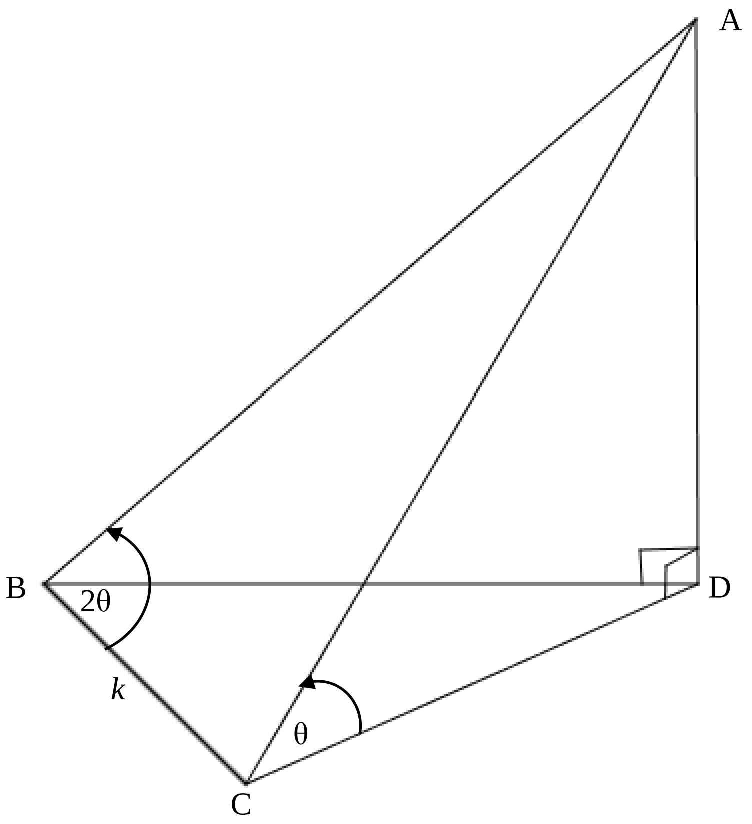
6.2.2 $f(x) = g(x)$, where $x \in [-90^0; 30^0]$? Show all relevant calculations. (6)

6.2.3 $f(x) > g(x)$, where $x \in [-90^0; 30^0]$? (2)

[11]

QUESTION 7

In the diagram below AD is a vertical structure. B, C and D are three points in the same horizontal plane. The angle of elevation of A from C is θ . BC = *k* metres, AB = 2BC and $A\hat{B}C = 2\theta$



7.1 Prove that $AC = k\sqrt{1 + 8\sin^2\theta}$ (5)

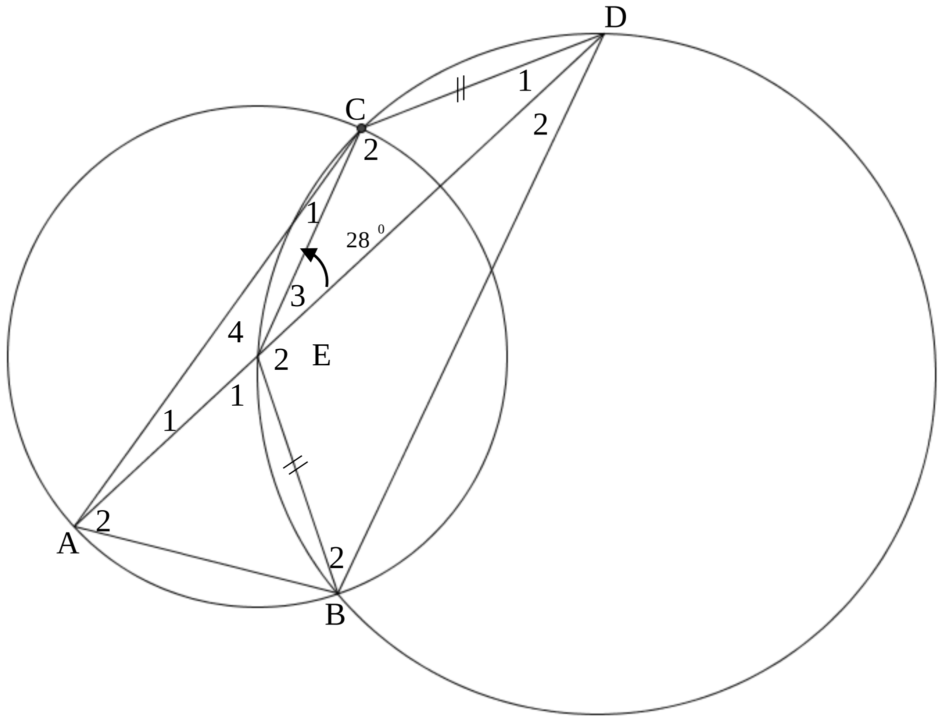
7.2 Calculate the value of AC rounded off to the nearest metre, if $k = 139{,}5$ metres and $\theta = 42^0$ (2)

[7]

Give reasons for your statements in QUESTIONS 8, 9, 10 and 11.

QUESTION 8

In the diagram below, E is the centre of the circle passing through A, B and C. AE is produced to D so that DCEB is a cyclic quadrilateral of the larger circle such that CD = EB. CA is joined. $C\hat{E}D = 28^0$



8.1 Determine the size of $\hat{D}_2$ (2)

8.2 Give a reason why $CE \parallel BD$ (1)

8.3 Show that CD = EC (2)

8.4 Calculate, with reasons, the size of:

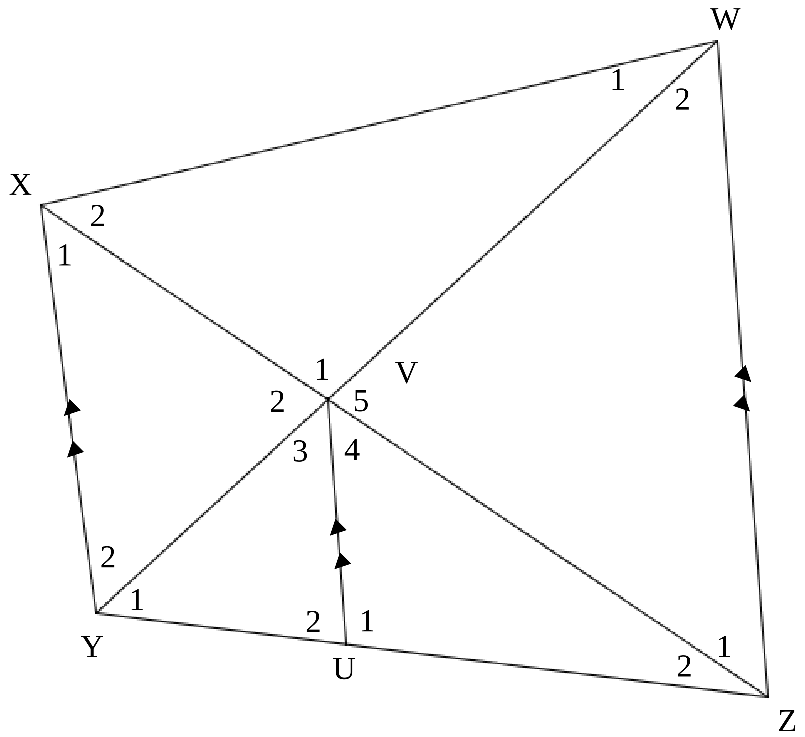
8.4.1 $\hat{B}_2$ (4)

8.4.2 $B\hat{A}C$ (3)

[12]

QUESTION 9

In the diagram below, WXYZ is a trapezium, with $XY \parallel UV \parallel WZ$. $UZ : YZ = 4 : 7$. $\hat{V}_3 = \hat{V}_4$



9.1 Prove that $\frac{YV}{VW} = \frac{XV}{VZ}$ (3)

9.2 Determine the numerical value of $\frac{\text{Area of } \Delta XVY}{\text{Area of } \Delta\ WVZ}$ in a simplified form. (4)

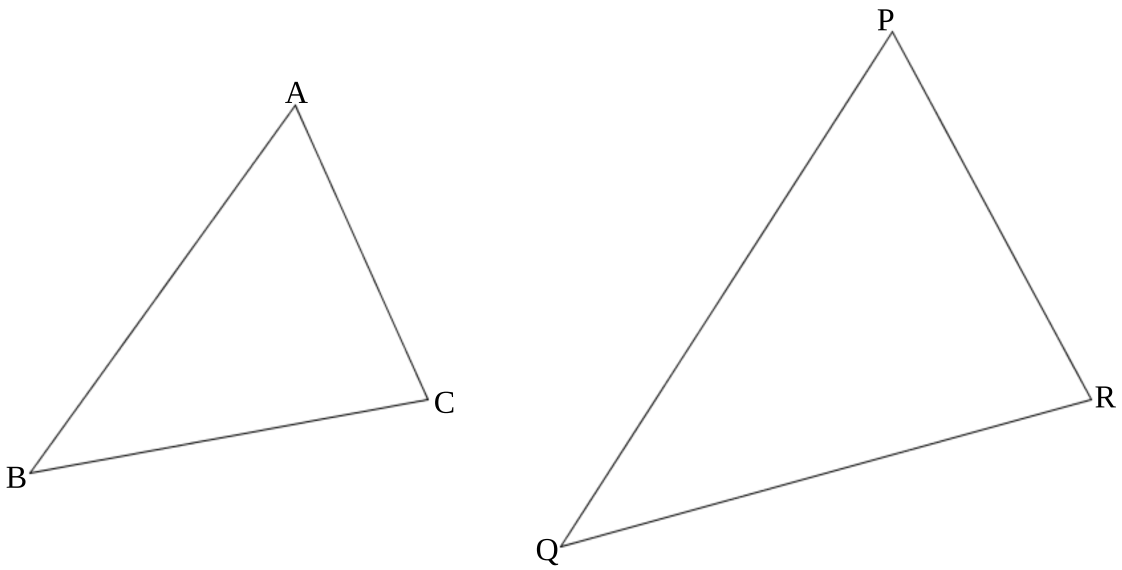
9.3 Show that WXYZ is a cyclic quadrilateral. (3)

9.4 Prove that UV is a tangent to the circle passing through X, V and Y. (2)

[12]

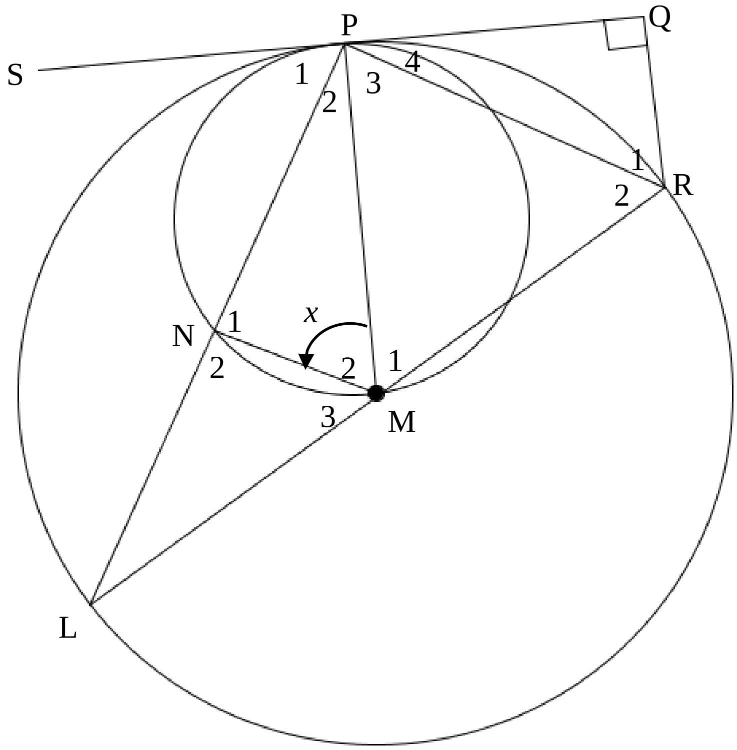
QUESTION 10

Complete the proof of the theorem in the answer book, that states that if in Δ ABC and Δ PQR, $\hat{A} = \hat{P}$, $\hat{B} = \hat{Q}$ and $\hat{C} = \hat{R}$.



10.1 Prove that $\dfrac{AB}{PQ} = \dfrac{AC}{PR}$ (6)

10.2 SPQ is a common tangent for both circles. M is the centre of the bigger circle. PM is the diameter of the smaller circle. $RQ \perp PQ$. $P\hat{M}N = x$



Prove that:

10.2.1 LN = NP (3)

10.2.2 PR bisects $Q\hat{R}L$ (4)

10.2.3 $\Delta PNM \parallel\!| \Delta PQR$ (4)

10.3 If it is given that, PR = 30 cm and QR= 15, calculate the:

10.3.1 length of LR. (5)

10.3.2 value of x (4)

[26]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1 + ni)$ $\quad$ $A = P(1 - ni)$ $\quad$ $A = P(1 - i)^n$ $\quad$ $A = P(1 + i)^n$

$T_n = a + (n - 1)d$ $\quad$ $S_n = \frac{n}{2}[2a + (n - 1)d]$

$T_n = ar^{n-1}$ $\quad$ $S_n = \frac{a(r^n - 1)}{r - 1}$; $r \neq 1$ $\quad$ $S_\infty = \frac{a}{1 - r}$; $-1 < r < 1$

$F = \frac{x[(1 + i)^n - 1]}{i}$ $\quad$ $P = \frac{x[1 - (1 + i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad$ $M\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad$ $y - y_1 = m(x - x_1)$ $\quad$ $m = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \tan\theta$

$(x - a)^2 + (y - b)^2 = r^2$

InΔABC: $\quad$ $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum x}{n}$ $\quad$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $\quad$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$